

3D GEOMETRY

UNIT A

SURFACE AREA

Name: _____

Topic	Assignment #	Date
Getting Ready (p. 3-4)		
Intro to 3D shapes (p. 5-9) <u>Area Formulas</u> (p. 8) Surface area – Nets (p. 10-17) Surface Area – Faces (p. 20)	1 (p. 18)	
<u>Surface Area formulas</u> (p. 21-22) Surface area Formulas: Prisms and cylinders (p. 23-24)	2 (p. 25-27)	
Surface Area Formulas: Pyramids, Cones and Spheres (p. 29-30)	3 (p. 31-33)	
Surface Area in Everyday life (p. 34-42)	4 (p.43)	
Info pages: Surface Area Formulas, Unit Conversion (p. 44-45)		

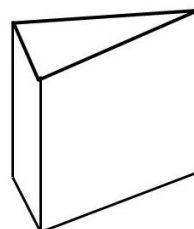
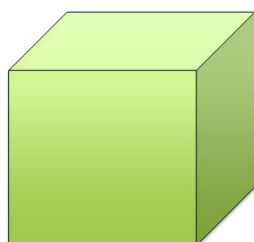
Geometry

Surface Area

- The total area of all the faces that make up a 3D shape

Volume

- The amount of space a 3D shape takes up.



Getting Ready

1) What are four SI (metric) units used to measure length? List them from shortest to longest.

2) Which SI units would you use to measure each item?



- a) the length of a salmon _____ b) the thickness of a coin _____
- c) the height of an apartment building _____
- d) the distance from Hopedale NL to Montréal QC _____

3) What are four imperial units used to measure length? _____

4) Which of these imperial units would you use to measure each item? a) the length of a cell phone _____

b) the diameter of a car tire _____ c) the width of a football field _____

d) the distance from Labrador City NL to Brandon MB _____



10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

5) Convert each SI length to the unit shown:

a) 4.5 m = _____ cm b) 275 millimetres = _____ centimetres

c) 1500 metres = _____ kilometres d) 1 m 80 cm = _____ cm = _____ m

6) Convert each imperial length to the unit shown.

Example: 6 ft = _____ inches

There are 12 inches in 1 foot.
So, 6 feet is $6 \times 12 = 72$ inches.

1 foot = 12 inches ($1' = 12''$)
1 yard = 3 feet

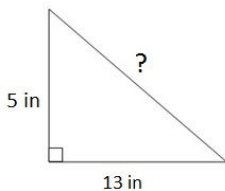
a) 42 inches = _____ feet b) 8 yards = _____ feet c) 10'6" = _____' = _____'

7) Determine the unknown side lengths in each right triangle, using Pythagoras.

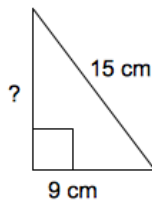
(Remember $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$)

-The two legs form the right (90°) angle ('the L'). -Hypotenuse is the longest side, opposite from the right angle)

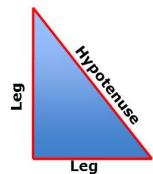
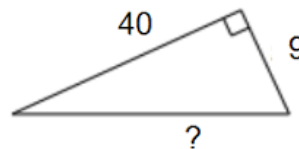
a)



b)



c)



8) Solving Equations

a) $P = 2(L + W) \rightarrow L = 5, w = 8$. Solve for p. b) $V = Lwh \rightarrow L = 7, w = 4, h = 2.5$. solve for V.

c) $A = 6s^2 \rightarrow s = 1.2$. solve for A.

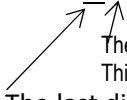
d) $A = \frac{bh}{2} \rightarrow A = 18, b = 3$. Solve for h.

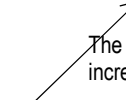
9) Rounding off

a) Decide which is the last digit to keep

b) Leave it the same if the next digit is less than 5 (this is called *rounding down*)

c) But increase it by 1 if the next digit is 5 or more (this is called *rounding up*)

Round 4.362 to the nearest unit. = 4

 The digit next to the 4 is a 3. Three is less than 5.
 This means leave the 4 as a 4.
 The last digit to keep is the 4. (don't keep the 3, the 6 or the 2)

Round to 3.567 to 1 decimal place = 3.6

 The digit next to the 5 is a 6. Six is 5 or more. This means
 increase the 5 to a six.
 The last digit to keep is the 5. (don't keep the 6 or the 7)

a) round 3.832 to: 1 decimal place _____ 2 decimal places _____ the nearest unit _____

b) round 4.952 to: 1 decimal place _____ 2 decimal places _____ the nearest unit _____

c) round 4.386 to: 1 decimal place _____ 2 decimal places _____ the nearest unit _____

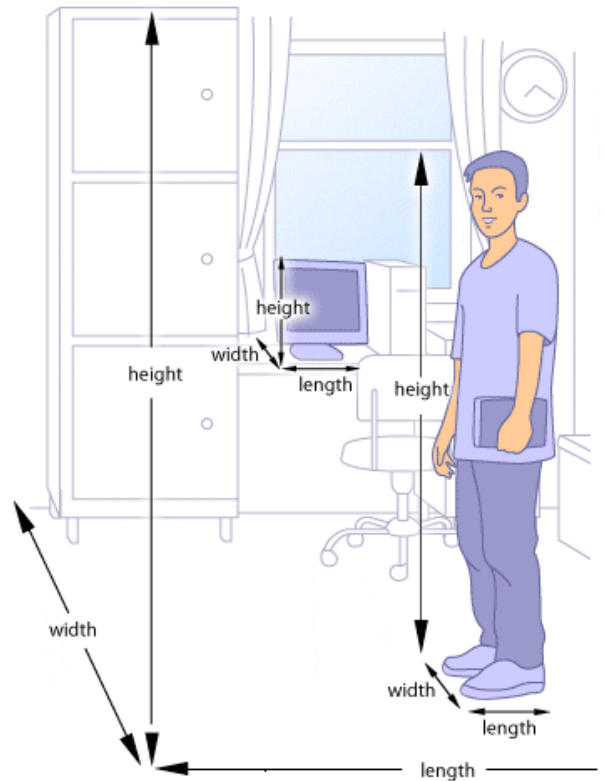
(Interactive Geometry – 3D shapes) – Go to site and look at interactive text explanation. Text is below.)

Introduction – 3D shapes

We live in a three-dimensional world. **Every object you can see or touch has three dimensions that can be measured: length, width, and height.** The *room* you are sitting in can be described by these three dimensions. The *monitor* you're looking at has these three dimensions. Even *you* can be described by these three dimensions. In fact, the *clothes* you are wearing were made specifically for a person with your dimensions.

In the world around us, there are many three-dimensional geometric shapes. In these lessons, you'll learn about some of them. You'll learn some of the terminology used to describe them, how to calculate their surface area and volume, as well as a lot about their mathematical properties.

This chapter focuses on the three dimensional, or 3-D geometry of prisms, pyramids, spheres, cylinders, and cones, including: surface area, volume and capacity, and making comparisons among the three ways of measuring 3-D objects.



3D Shapes

There are many types of three-dimensional shapes. You've surely seen spheres and cubes before. In this chapter, you'll learn about three-dimensional shapes whose faces are **polygons** (*triangles, squares, circles, etc.*) — and you'll also learn about three special types of 3-D shapes: **prisms, pyramids (including cones which are circular pyramids), and spheres.**

Prisms – the **top and bottom** of a prism is the **same shape and size** (congruent polygons). These two sides are opposite each other and parallel to each other and are the **bases** of the prism. The faces that are not the bases are called lateral faces. They connect the two bases. The **lateral faces** are usually rectangles. (*Remember a square is a special rectangle with length and width equal, or congruent.*)

Note: the base is not necessarily the bottom of the prism - it's the shape that's consistent throughout the entire object. The shape of the base determines the name of the prism.



Examples of Prisms: The shape of the base can determine the name of the prism. The faces that are not the bases are called **lateral** faces.

A cereal box is in the shape of a **rectangular prism**.

- The **bases** of a rectangular prism are 2 congruent **rectangles**.
- The **lateral faces** are **two more pairs of congruent rectangles**. (*There are 3 pairs of rectangles: top and bottom are the same; front and back are the same; the two sides are the same.*)

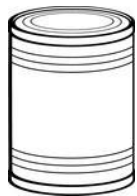


A **die** is in the shape of a **cube**. A cube is a *special rectangular prism* where all the faces are the same.

- The **faces** of a cube are **6 squares that are all the same**.

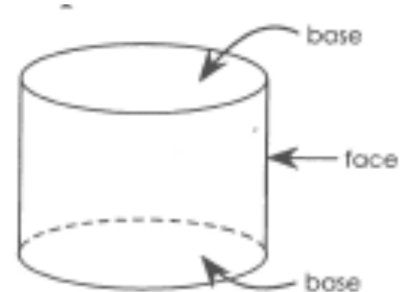
You can buy a **block of cheese** in the shape of a **triangular prism**.

- The **bases** of a triangular prism are **two triangles** that are the same (*top and bottom*).
- The **lateral faces** are **3 rectangles** (*sides*).



A **can of soup** is in the shape of a **cylinder**. Its faces are **two circles and a rectangle** (*Think of the paper label wrapped around the can – if you peel it carefully off the can, it's a rectangle.*).

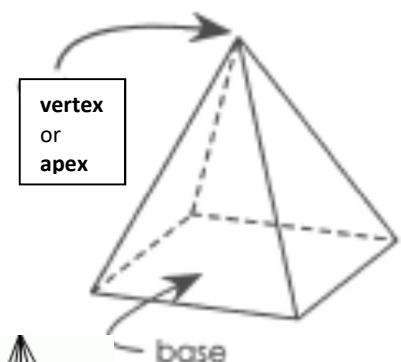
- **A cylinder is a prism, but because it has a round base, it has only one rectangular face that wraps around the two circular ends to connect them.**



Pyramids:

Of course the **pyramids in Egypt** are in the shape of a pyramid!

- **A pyramid is an object in which the shape of the base reduces to a single point throughout the height of the object.**
- The **base** of a **pyramid** is a **polygon** (square, pentagon, triangle, etc.), and all **lateral faces** are **triangles that share a common vertex**. Often all the triangles (lateral faces) are congruent – ie they're all the same size and shape.
- Similar to prisms, the shape of the base determines the name of the pyramid.



Cone or Circular
Pyramid



Triangular
Pyramid

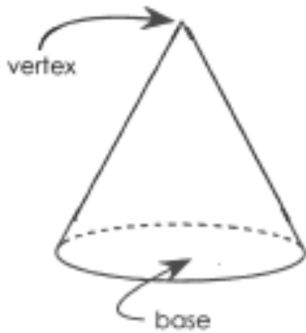


Rectangular
Pyramid



Pentagonal
Pyramid

Cone:



The **pointy cone** you can choose for an **ice cream cone** is in the shape of a cone.

- Similar to a cylinder, a **cone** is a **form of a pyramid with a round base**.
- As a result of the round base, instead of having multiple triangular lateral faces (like a pyramid), a cone **has one lateral face that wraps around the circular base and comes to a point (or vertex)**.

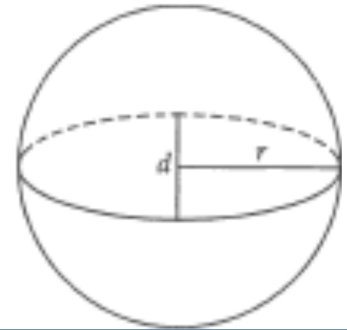


Sphere:



A ball is a **sphere**.

- A sphere is a 3-D ball-shaped object in which **all points are equidistant (the same distance) from the centre**. (The distance from the centre is the **radius** of the sphere.)



Introduction – Surface Area and Volume



Have you ever wrapped a birthday gift? If so, then you've covered the **surface area** of a prism with wrapping paper.

Have you ever poured yourself a glass of milk? If so, then you've filled the **volume** of a glass with liquid.



Surface area is exactly what it sounds like — **the area of all of the outside surfaces of a three-dimensional object**.

And **volume** is all of the **space inside a three-dimensional object**.

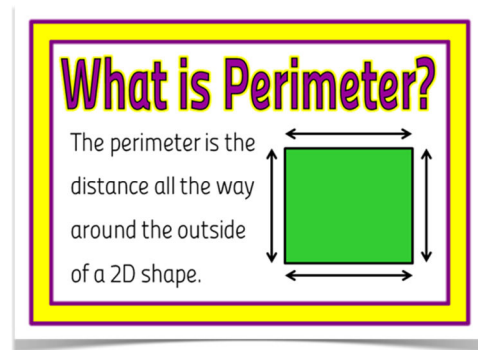
Surface area is often used in construction. If you need to paint or any 3-D object, you need to know **how much paint to buy**. If you're putting up drywall you need to know **how much material** to buy. Or when wrapping a gift, the **amount of wrapping paper** needed to cover the figure represents its surface area.

Lesson 1: Surface Area

In this lesson, you will:

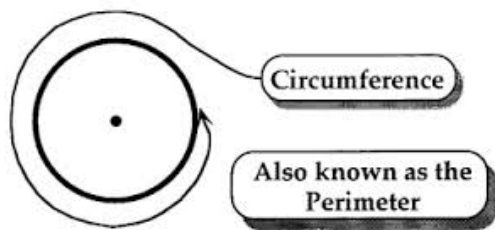
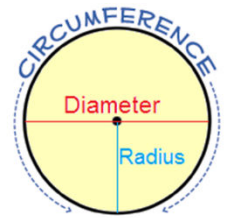
- Observe how area and surface area are related
- Review the characteristics of prisms, pyramids, and spheres
- Use 3 methods to calculate the surface area of 3-D objects: nets, the faces approach, and formulas.
- Calculate the surface area of composite 3-D objects
- **Measuring Surface Area.**

Review of Gr.10 Geometry



- **Perimeter** is the distance around the outside of a 2-D shape or object. To calculate it, add the lengths of all sides.

Perimeter (called circumference) of a **circle** has a special formula, which is $C=2\pi r$ or $C=\pi d$. The formulas are the same because the **radius** of a circle is half the diameter, or twice the radius is the **diameter**.



$$C = 2\pi r = \pi d$$

$$C = 2\pi r \quad \text{Use when you know the radius.}$$

$$C = \pi d \quad \text{Use when you know the diameter.}$$

remember...
radius = $\frac{1}{2}$ diameter
diameter = $2 \times$ radius

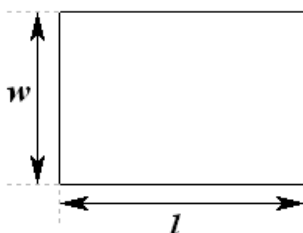
You may use 3.14 as the value of π or you can use the π button on your calculator.

Area refers to the **number of square units needed to cover the surface of the interior region** of a 2-D geometric shape. Each shape has its own formula for calculating area.

To understand area, it may help to think of how much paint would be needed to completely cover the shape.

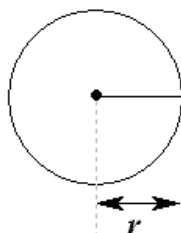
To be able to calculate the areas of the 2-D figures, we must remember the area formulas.

Rectangle:



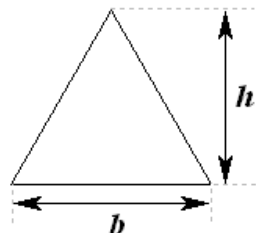
$$\text{Area} = lw$$

Circle:



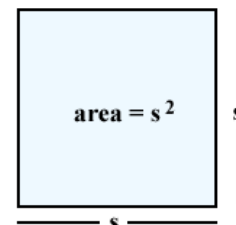
$$\text{Area} = \pi r^2$$

Triangle:



$$\text{Area} = \frac{1}{2}bh$$

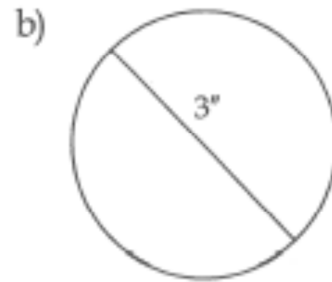
Square:



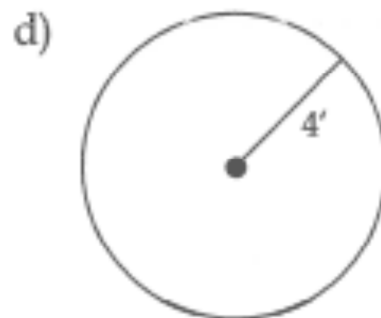
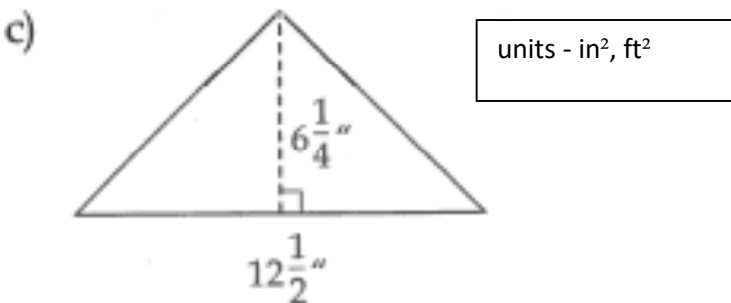
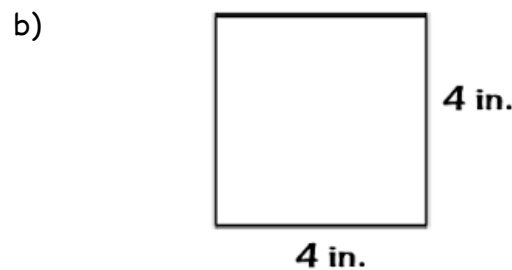
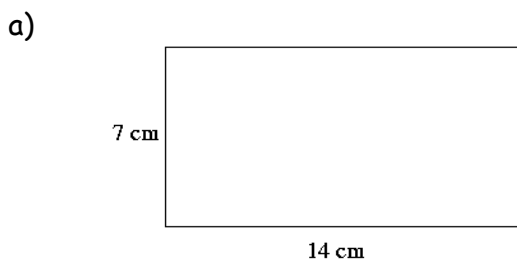
A square is just a rectangle with the length and width equal.
 $L \times w = s \times s = s^2$

EXAMPLES: Use the formulas given on p. 8 and at the end of this booklet to calculate the area, perimeter, and circumference, and areas of the following shapes. Round off to 1 decimal place when needed.

1. Find the **perimeter** of the following figures. Include the units.



2. Find the **area** of each of the following.

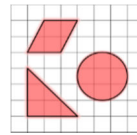


(It is likely easier to convert 6 $\frac{1}{4}$ and 12 $\frac{1}{2}$ to decimals.
If you don't know the decimal for a fraction,
divide **numerator (top) ÷ denominator (bottom)**).

1a) 16m b) 9.4" 2a) 98 cm²,
b) 16 in², c) 39.1 in², d) 50.3

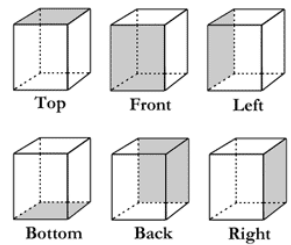
Area of 2D shape vs Surface area of 3D shape

Area is the amount of space taken up by a **2-D object**.



Surface area is the area of the **entire surface** of a **3-D object**. You can use your knowledge about the area of two-dimensional shapes to calculate the surface area of a three-dimensional shape, since each face or side is effectively a two-dimensional shape.

Surface Area of a Prism



You therefore work out the **area of each face**, and then **add them together** to find the **total area of all the faces** of the object. This means that the units for surface area are the same as the units for area (**units²**, like m², or in²). You will learn three different ways to calculate surface area in this lesson: **nets**, the faces approach, and formulas.

Method 1: Finding Surface Area using Nets

The surface area of a regular prism is equal to the **sum of the area of all of its faces**. So imagine if a 3-D figure were made out of cardboard and you were able to cut it to open it up **flat** .. still leaving it connected. When you open it up, it's easier to visualize all the 2-D faces so you can calculate their areas. We call this a **net**.

A net is a flat (2-D) diagram (a flattened out picture) of all the faces of a 3-D object.

The **surface area** is the **total area covered by the net** of a 3-D shape.

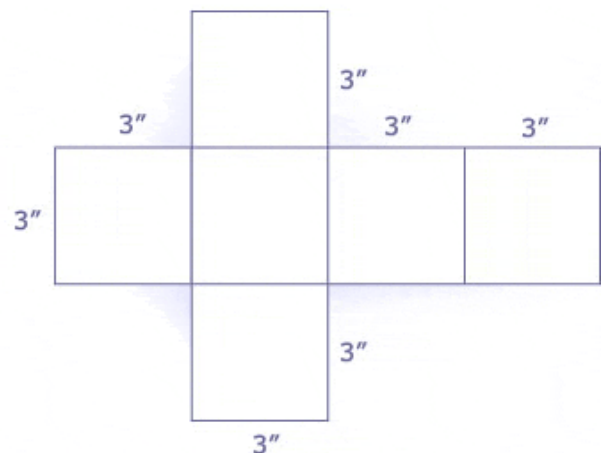
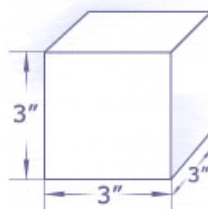
To find the surface area, calculate the area of each of the faces and then add up all the areas for the total surface area. We can make nets for objects with straight sides like prisms (but not for cylinders, cones, or spheres).

To draw a net, you must imagine that you are **unfolding** the object.

Each face of the object must still be **touching** at least one of the faces with which it shared an edge in the 3-D version. The **order** of the faces that form a straight line in a net must be the **same** as their 3-D order.

Example: Let's take a look at a cube.

As you already know, a cube has **six square faces**. If each of those faces is **3 inches by 3 inches**, then the **area of each face is $3 \times 3 = 9 \text{ in}^2$** . And since there are **six** of them, the total surface area is **$9 + 9 + 9 + 9 + 9 + 9$ (or 6×9) = 54 in^2**



For help visualizing nets, go to this link.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object_interactives/surfaceArea/use_it.html

Do the “use it” activity. For each 3-D shape, move the slider back and forth several times to view the animation of the net for the object. Pay close attention to where each face goes when open and which parts are touching. After watching the animation several times, choose the net for the object (don’t look at the animation when choosing!). Note the differences between the nets to help you decide which to choose.

To find the surface area of many regular 3D shapes using a net, you can follow the process described below:

1. Draw (or imagine) a net of the prism.
2. Calculate the area of each face.
3. Add up the area of all the faces.

Special Case for Net of Cylinder.

You will draw or imagine two circles and one rectangle (think of a soup label carefully peeled off a tin).

Find the area of the two circles and add together. Then add the area of the rectangle:

The **circumference** of the circle is $2\pi r$.

The **height** h of the rectangle is equal to the height of the cylinder.

The **base length** of the rectangle b is equal to the circumference of the cylinder. $2\pi r$

Area of rectangle of cylinder

= **Length of base of rectangle times height of rectangle**

= circumference **times** height

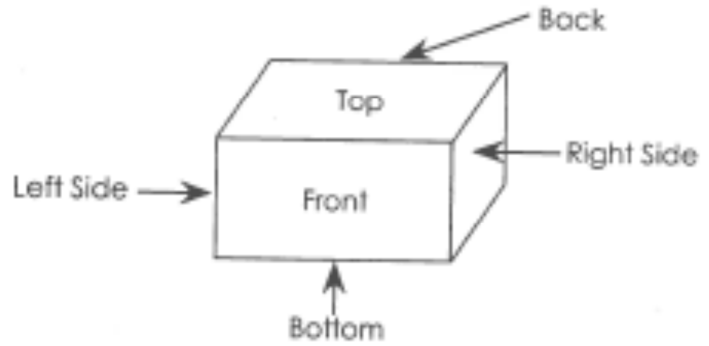
= $(2\pi r)(h)$

= $2\pi rh$

Materials required: net; scissors; pencil, ruler, eraser, tape

Example 1: - surface area with nets

- a) With your group, cut your net out carefully. Cut the net along the edges and be sure to keep each face connected to at least one other face. The net should still be in one piece.



Fold along the lines to form a box (a rectangular prism). (use a little bit of tape if needed to hold together, but not too much - you need to take apart again). Label each face (front, back, top, bottom, left side, right side).

- b) Do your best to sketch the net of this box below without cutting the box apart (don't include the tabs in your net). (Each person sketches their own net.)

The drawing does not need to be the actual size of the box and does not need to be to scale. You should be able to tell the difference between squares and rectangles, and between different sizes of rectangles. Equal sides should be drawn approximately the same length. Label the sections according to the faces you think they represent (front, back, top, bottom, left side, right side).

- c) Open your box back up to produce a net (labelled side up). You should be able to lay the cut-up box flat on the table. The net should contain 6 rectangles. Does the net look approximately similar to the net you drew? If, not make changes to your sketch of the net.

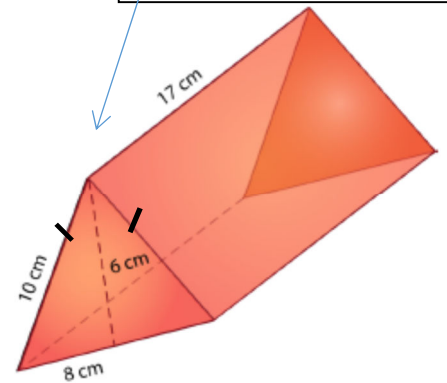
Measure the length of each side of the rectangles (not the tabs) and label the dimensions them on your net (include the unit - centimetres).

d) **Calculate the area of the 3 pairs of rectangles.** For each face, write the area formula; substitute the values into the formula; simplify.

e). **Add the faces together for the surface area of the object. Include the units, squared.** (173.9 cm²)

Example 2: Use the net of the triangular prism to calculate the surface area.

To find the surface area, we must be able to calculate the area of each face and then add these areas together. One way to do this is to use a net. *Remember that a net is a two-dimensional representation of a three-dimensional solid. A net is a stretched out picture or an unfolded picture of a solid.* If we look at a net and find the area of each surface of the net and then add up (find the total of; find the sum of) all the areas then we will know the measurement of the “cover” of the figure. What is the surface area of the triangular prism?



1. Draw a net: Get ready to exercise your imagination! It may help to shade the top and bottom faces to keep you on track.

a) Begin by drawing the bottom face. It is a triangle.

b) Each side of the face is connected to a side face. What shape is each side face? They are rectangles, so we draw rectangles along each side of the triangular base. Draw the faces that share an edge with the face you drew in (a). Draw this face so that it shares a side with the face from (a).

c) Lastly, we draw the top face, which can be connected to any of the side faces. Be sure that you do not draw the same face twice!

2. Fill in the measurements for the sides of each face so that we can calculate their area. Be careful!

(This time two of the faces are triangles. Remember, we calculate the area of triangles with the formula

$A = \frac{bh}{2}$. Therefore we need to know the height of the triangles. Look at the diagram to find it.

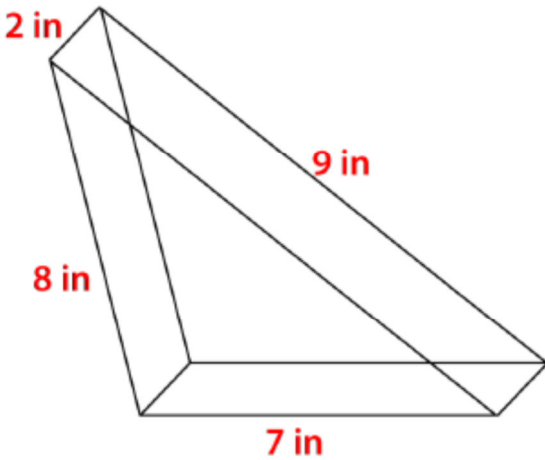
3. Find the area of each 2-D face. This time we are going to find the areas of two triangles and three rectangles. **For each face, write the area formula; substitute the values into the formula; simplify.**

4. Add the faces together for the surface area of the object. Include the units, squared.

Try it! Find the surface area of this triangular prism, using the net method.

Draw the net of this triangular prism. Find the area of each face. For each face, write the area formula; substitute the values into the formula; simplify. Add the faces together for the surface area of the object. Include the units, squared.

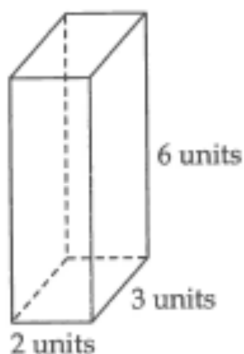
(104 m²)



Try it! Find the surface area of this rectangular prism, using the net method.

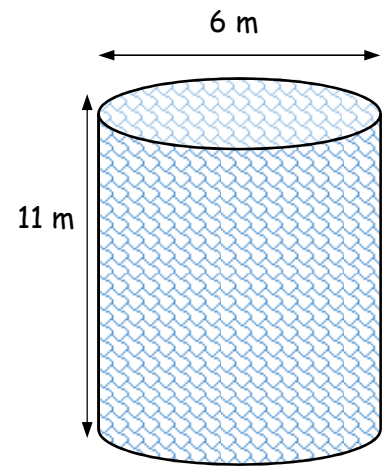
Draw the net of this rectangular prism. Find the area of each face. For each face, write the area formula; substitute the values into the formula; simplify. Add the faces together for the surface area of the object. Include the units, squared.

(72 units²)



Example 3: Use the net of this cylinder to calculate surface area.

a) Draw the net of the cylinder to the right



b) Calculate the surface area of the cylinder. Find the area of each face. **For each face, write the area formula; substitute the values into the formula; simplify.** Add the faces together for the surface area of the object (round off to 1 decimal place). Include the units, squared.

(263.9 m²)

Remember the trick to finding the lateral area (the rectangle – the unpeeled soup label) from p. 9

To find lateral area (L.A.):

Take a can of soup
Peel off the label
Multiply **circumference** by height



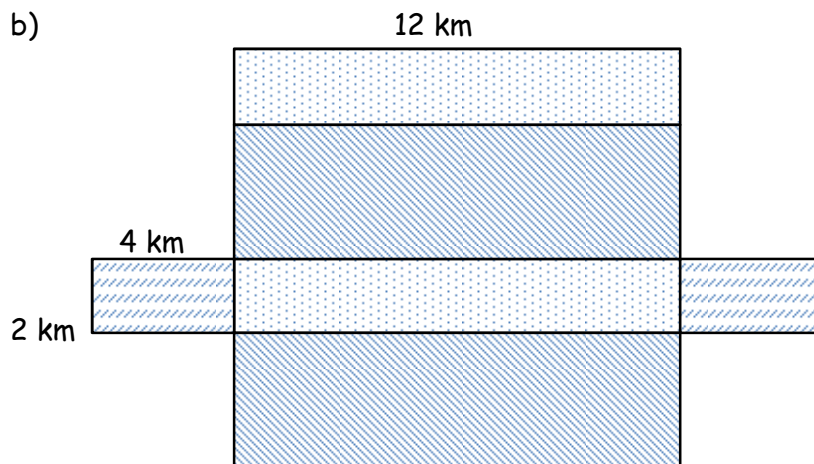
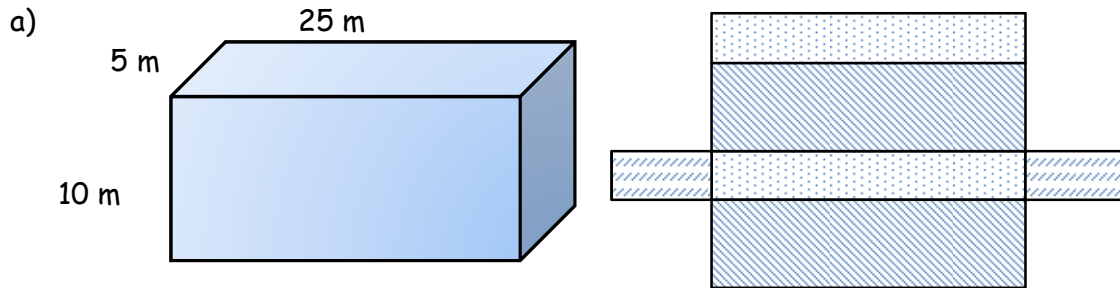
height

circumference

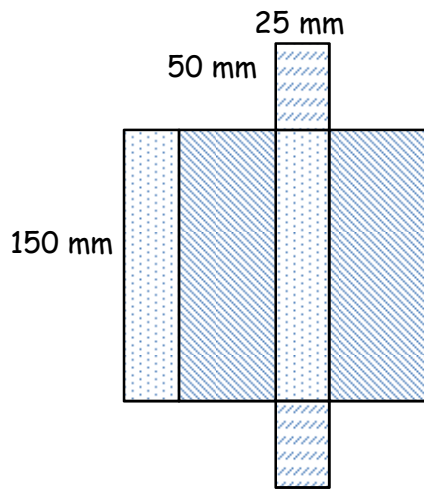
Assignment 1: Surface Area and nets (Rectangular Prisms & Cylinders)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final **answer** *

1. Label the nets of the figures below and determine the *surface area*. (solutions next page)

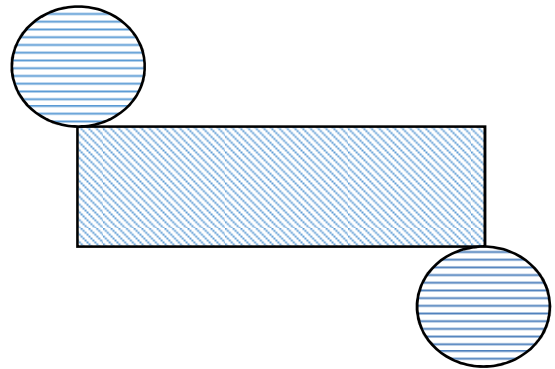
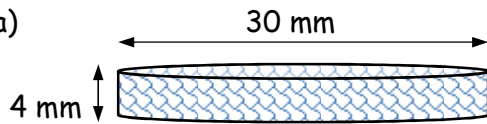


c)

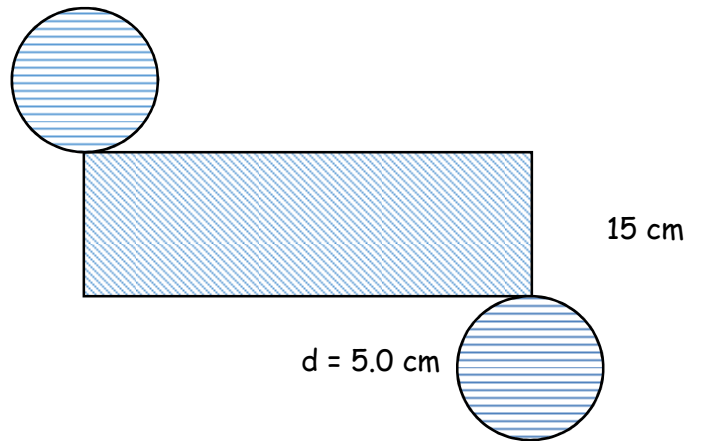


2. Using the net, determine the *surface area* of the following **cylinders**: (round off to 1 decimal place)

a)



b)



Answers: 1) a) 850m^2

b) 160 km^2

c) 25000 mm^2

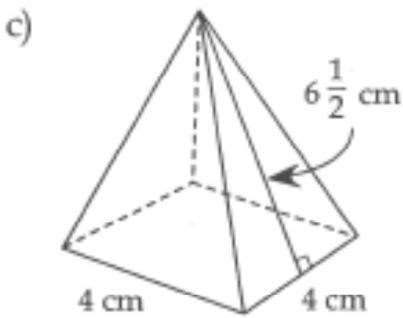
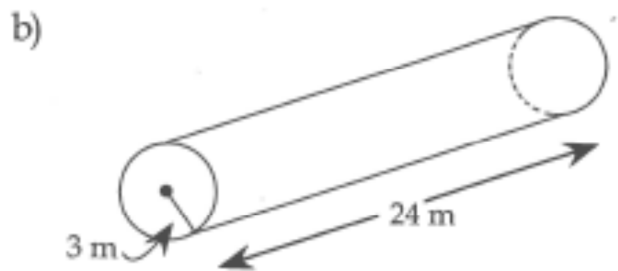
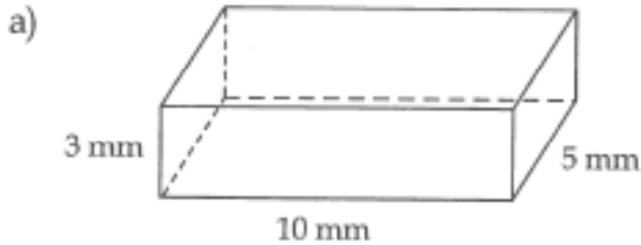
2) a) 1790.7 mm^2 b) 274.9 cm^2

Method 2: Finding Surface Area *using the Faces* of the Object

Calculate the area of each face of the object, and then add all these areas together. We call this method the **faces approach**. *It is like the net method, without drawing the net.* You can visualize the net in figuring out what faces make up the object, and their dimensions, so that you can calculate their areas. Then you add up the area of the faces for the total surface area.

For the following, name the type of 3-D object, and **find the surface area** using the faces approach.

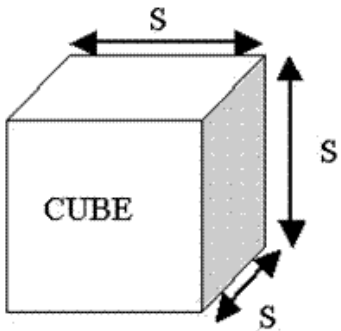
a) rectangular prism 190 mm^2 b) cylinder 508.9 m^2 c) squared based pyramid 68 cm^2 (↓ for b, round off to 1 decimal place)



Method 3: Finding Surface Area using Formulas

For many 3D objects, like prisms, pyramids and spheres, there are general formulas that can be used to find the total surface area. These formulas show a pattern that was found when solving for surface area of specific objects.

Cube



•Surface Area

Area of one square $A = s^2$

There are 6 squares.

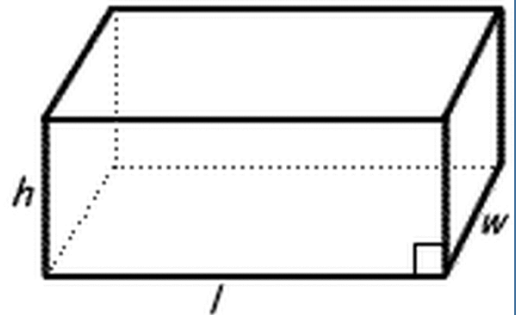
Therefore formula $A = 6s^2$

•Volume $V = s^3$

Rectangular Prism

Surface Area

$$A = 2wh + 2lw + 2lh$$



Volume

$$V = lwh$$

Cylinder

Surface Area

We will need to calculate the surface area of the top, base and sides.

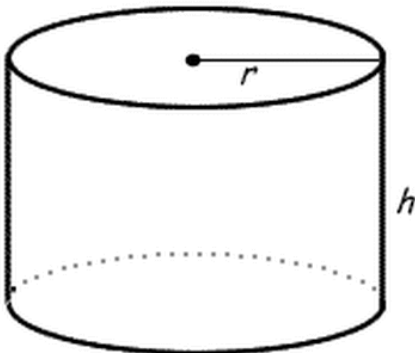
Area of the top is πr^2

Area of the bottom is πr^2

Area of the side is $2\pi rh$

Therefore the Formula is:

$$A = 2\pi r^2 + 2\pi rh$$

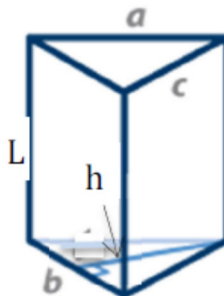


Volume

$$V = \pi r^2 h$$

b – edge b (base of triangle)
h – height of triangle
L – length (height) of PRISM
a – edge a
c – edge c

Volume $V = \left(\frac{bh}{2}\right)L$



Triangular Prism

•Surface Area - 2 triangles 3 rectangles

(If base is isosceles triangle then 2 rectangles have the same area)

$$A_{\text{base triangles}} = 2\left(\frac{bh}{2}\right)$$

$$A_{\text{3 rectangles}} = aL + bL + cL$$

$$A_{\text{total}} = 2A_{\text{base triangles}} + A_{\text{3 rectangles}}$$

Therefore the formula is:

$$A = 2\left(\frac{bh}{2}\right) + aL + bL + cL$$

Cone

Surface Area

We will need to calculate the surface area of the cone and the base.

Area of the cone is πrs

Area of the base is πr^2

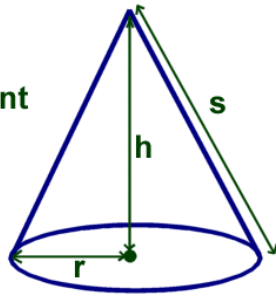
Therefore the Formula is:

$$SA = \pi rs + \pi r^2$$

r = radius

h = height

s = length of slant



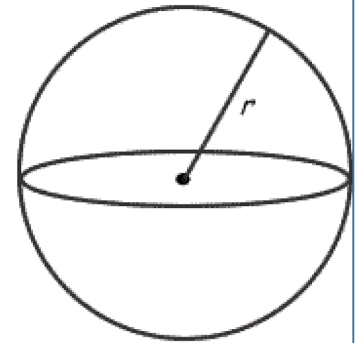
Volume

$$V = \frac{1}{3} \pi r^2 h$$

Sphere

Surface Area

$$A = 4 \pi r^2$$



Volume

$$V = \frac{4}{3} \pi r^3$$

- The surface area of a sphere is four times the product of π and the square of the radius.

$$SA = 4\pi r^2$$

- To understand this formula, think about a baseball covering.

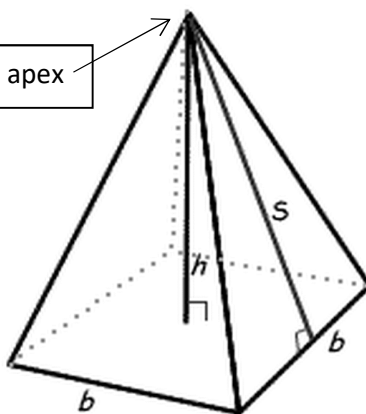


Square Based Pyramid

Surface Area

$$SA = b^2 + 2sb$$

apex



Volume

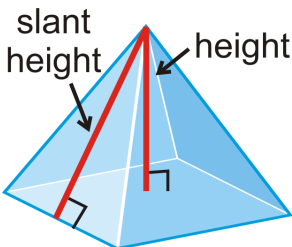
$$V = \frac{1}{3} b^2 h$$

(s - slant height - height of triangle
 h - vertical height from apex to base of pyramid)

area of square base = b^2 ;

area of 4 triangles = $4\left(\frac{bs}{2}\right)$,

which simplifies to $2bs$



Vocabulary of 3D shape:

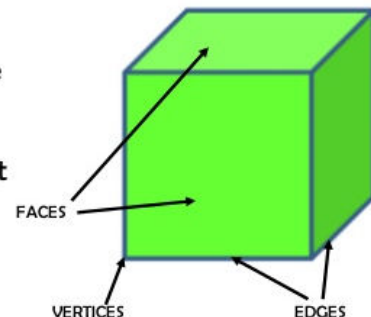
FACES, VERTICES and EDGES

3D shapes can be described in 3 ways:

Faces - the sides of the shape

Vertices - the corners

Edges - where the faces meet



Method 3: Surface Area (continued)

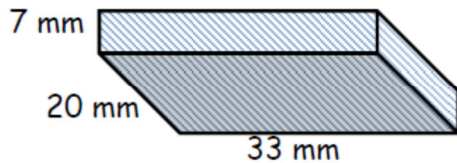
A) Prisms & Cylinders (using formulas)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final **answer** *

Example 1: Determine the total *surface area* of the following **prisms** using the FORMULA on p. 21-22 or back of this booklet.

a) Rectangular Prism (2062 mm²)

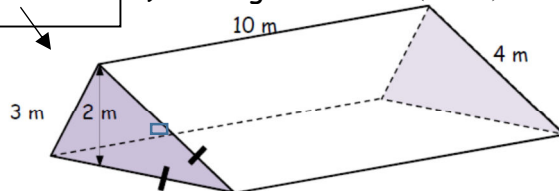
$SA_{\text{rectangular prism}} =$ _____



triangle is isosceles
– two sides equal
means two
rectangles are equal

b) Triangular Prism (118 m²)

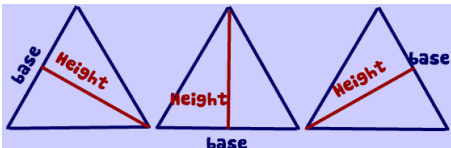
$SA_{\text{triangular prism}} =$ _____



area of triangle

$$A = \frac{bh}{2}$$

h – height of triangle
perpendicular (90°)
to base



3 rectangles – picture each separately
and find Lw for each one

Example 2: Determine how much aluminum is needed to make the following container
(rounded to first decimal place)? 384.9 cm²

(Hint: find the *surface area* of the following **cylinder**. Use π button or 3.14)



$$SA_{\text{cylinder}} = \underline{\hspace{10cm}}$$

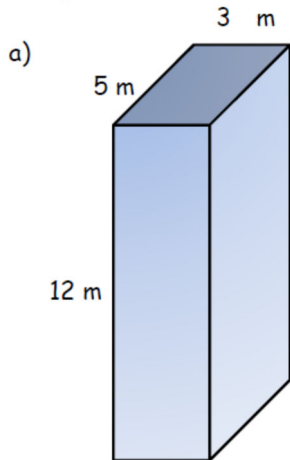
Assignment 2:

Surface Area (Prisms - rectangular, triangular & Cylinders)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

(solutions p. 28)

- 1) Using the formula for rectangular **prisms**, determine the total *surface area* of the following: (formulas p. 18)



$$L = \underline{\hspace{2cm}} \quad W = \underline{\hspace{2cm}} \quad H = \underline{\hspace{2cm}}$$

$$SA = \underline{\hspace{4cm}}$$

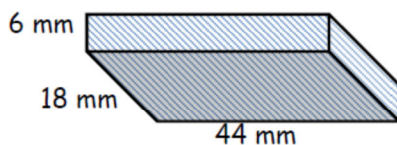
$$SA = 2 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + 2 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + 2 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$

$$\hspace{10em} (w) \hspace{10em} (h) \hspace{10em} (L) \hspace{10em} (w) \hspace{10em} (L) \hspace{10em} (H)$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{4cm}}$$

b)



$$L = \underline{\hspace{2cm}} \quad W = \underline{\hspace{2cm}} \quad H = \underline{\hspace{2cm}}$$

$$SA = \underline{\hspace{4cm}}$$

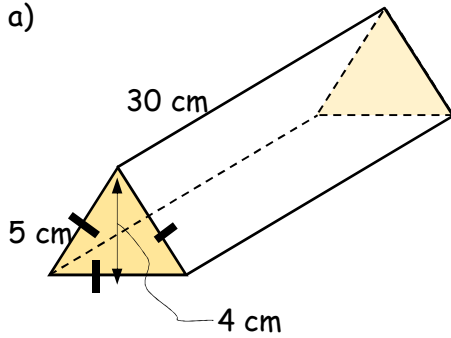
$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{4cm}}$$

2) Determine the total surface area of the following **prisms (triangular)**: (formula p. 19)

a)



$$A = 2\left(\frac{bh}{2}\right) + aL + bL + cL$$

$$B = \underline{\hspace{2cm}} \quad h = \underline{\hspace{2cm}}$$

$$SA = (2 \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \div 2) + (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) + (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) + (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$$

B
 H

a
 L

b
 L

c
 L

right
rectangle

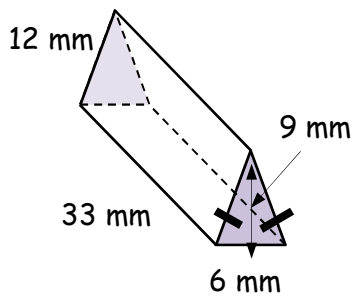
left
rectangle

bottom
rectangle

$$SA = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{4cm}}$$

b)



$$B = \underline{\hspace{2cm}} \quad h = \underline{\hspace{2cm}}$$

$$SA = \underline{\hspace{4cm}}$$

$$SA = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

right

left

bottom

$$SA = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{4cm}}$$

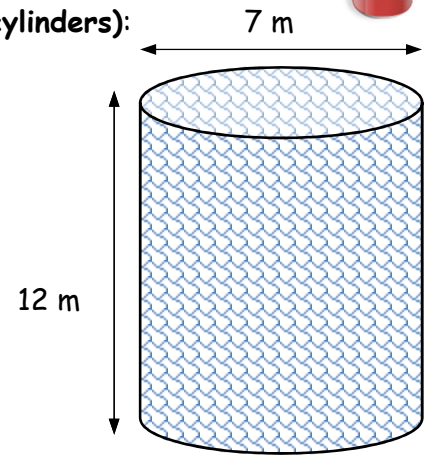


3) Determine the total surface area of the following prisms (cylinders):

a) $SA = 2\pi r^2 + 2\pi rh$

$r = \frac{\text{diameter}}{2} =$ _____

$h =$ _____



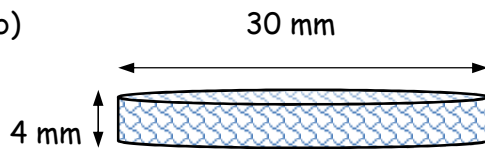
$SA = [2 \times \pi \times (\text{_____})^2] + [2 \times \pi \times \text{_____} \times \text{_____}]$

$r \qquad \qquad \qquad r \qquad \qquad \qquad h$

$SA =$ _____ $+$ _____

$SA =$ _____

b)



$r = \frac{\text{diameter}}{2} =$ _____ $h =$ _____

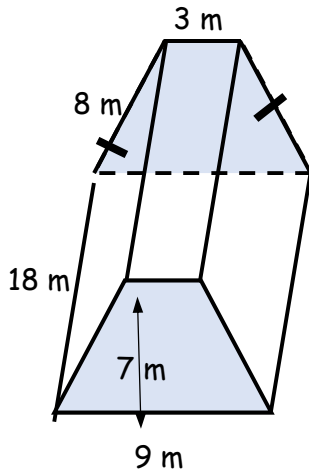
$SA =$ _____

$SA =$ _____ $+$ _____

$SA =$ _____ $+$ _____

$SA =$ _____

- 4) Determine the total surface area of these **trapezoidal** prisms. (BONUS)
(hint: see formula for area of a trapezoid at the end of this booklet.)



a) What 2D shapes make up the faces?

- two _____
- four _____

b) one trapezoid: $a = \underline{\hspace{1cm}}$ $b = \underline{\hspace{1cm}}$ $c = \underline{\hspace{1cm}}$
(write formula, substitute in values, solve)

c) 2 trapezoids: $2(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

d) Find area of 4 rectangles and add together:

$$\begin{array}{ccccccc} \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} & + & \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} & + & \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} & + & \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \\ \text{(side)} & & \text{(side)} & & \text{(top)} & & \text{(bottom)} \end{array}$$

$$= \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

e) surface area of trapezoidal prism = area of 2 trapezoids + area of 4 rectangles

$$= \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

Answers:

1) 222 m^2 , 2328 mm^2

3) 340.9 m^2 , 1790.7 mm^2

2) 470 cm^2 , 1044 mm^2

4) 588 m^2 ,

Lesson 3: Surface Area (continued)

B) Pyramids, Cones and Spheres

Example 1: The entrance to the Louvre Museum in Paris, France is a giant glass squared-based pyramid. It has a square base with **sides 35m long** and a **slant height of 10m**. Find out how much glass is needed to cover the surface and floor of the Louvre (determine the total *surface area*).

$SA = b^2 + 2sb$ (see explanation of formula p. 22)

(1925 m²)

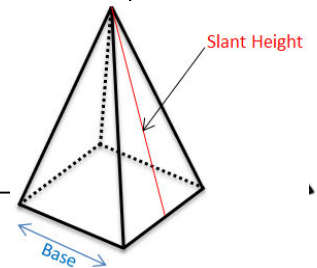


SA = area of base square) + area of 4 triangles

$$SA = b^2 + 2sb$$

S = **slant height** (height of triangle) = ____

b = **base** of triangle; side of square = ____



SA =

Example 2: A red cedar statue of a trout stands outside the Chamber of Commerce in Kamloops, B.C. The stand is a metal **cone** with **radius 1.6 m** and **slant height 3m**. What area of metal (rounded off to 1 decimal place) is needed for the stand and its hidden base? (23.1 m²)

(see p. 22 for explanation of formula)



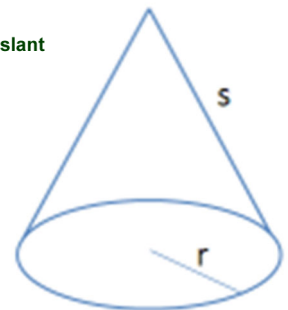
$$SA = \pi r^2 + \pi rs$$

$$r = \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{2cm}}$$

SA =

r = radius
h = height
s = length of slant

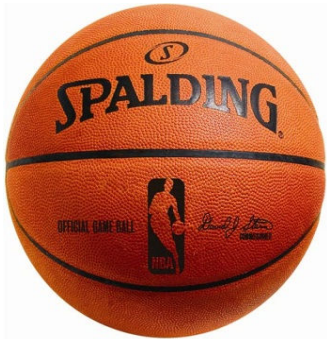


Use x^2 and π buttons on calculator

Round off to 1 decimal place

Example 3 : Since Beatty is such an awesome basketball player, she wants to make an official NBA basketball. An official ball has a diameter of 9.5 in. How much leather will she need for the surface (rounded to 1 decimal place)? (see explanation of formula p. 22) (283.5 in²)

(i.e. Determine the surface area of the **sphere**)



$r =$ _____

$$SA = 4\pi r^2$$

Use x^2 and π buttons on calculator

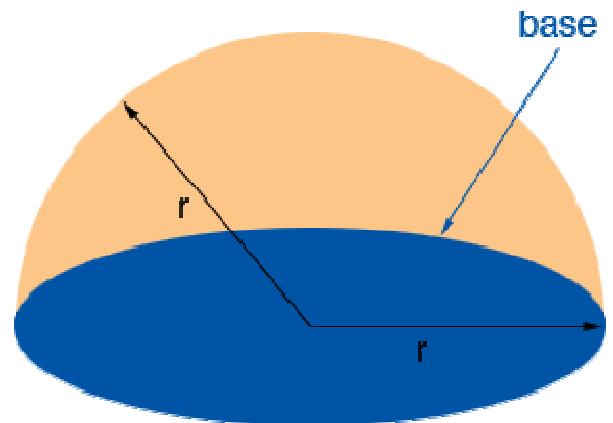
Round off to 1 decimal place

Example 4 : Half of a sphere is called a **hemisphere**. If the formula for the surface area of a sphere is $4\pi r^2$, then the surface area of a half a sphere is

$(4\pi r^2) \div 2$, or $\frac{4\pi r^2}{2}$ which equals _____.

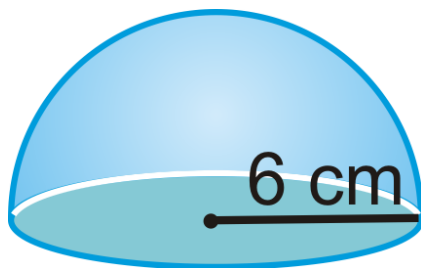
But the hemisphere also has a base, which is a **circle**.
The formula of the area of a **circle** is _____.

Therefore the formula for the surface area of a hemisphere is: _____.



Hemisphere: half a sphere plus base (circle of radius r)

Find the surface area of the following hemisphere with radius 6 cm, rounded to 1 decimal place. (339.3 cm²)

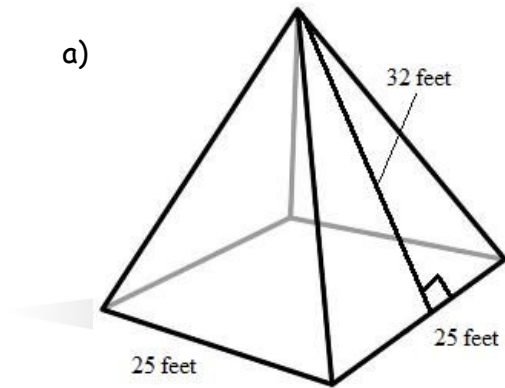


Assignment 3: Surface Area (Pyramids, Cones & Spheres)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

1) Determine the total *surface area* of the following **pyramids**:

a)



$$b = \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{2cm}}$$

$$SA = b^2 + 2sb$$

Use x^2 button on calculator

$$= (\underline{\hspace{2cm}})^2 + [2 \times (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}})]$$

b

b

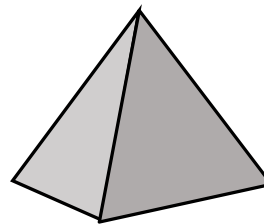
s

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

b) Square Base length = 7 cm, slant height = 5 cm

$$SA = \underline{\hspace{2cm}}$$



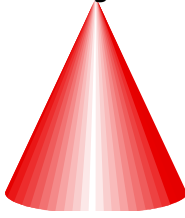
$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

2) Determine the total surface area of the following **cones**, rounded to 1 decimal place:

a) slant height = 35 cm



Dia. = 20 cm

$$SA_{\text{cone}} = \pi r^2 + \pi r s$$

$$\text{Radius (r)} = \frac{\text{Diameter}}{2} = \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{2cm}}$$

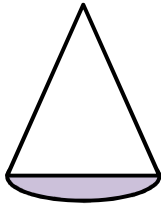
$$SA = \pi \times \left(\frac{\hspace{1.5cm}}{r} \right)^2 + \pi \times \frac{\hspace{1.5cm}}{r} \times \frac{\hspace{1.5cm}}{s}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

round to 1 decimal place

b) Slant height = 70 mm, radius = 50 mm



$$SA_{\text{cone}} = \underline{\hspace{2cm}}$$

$$SA = \pi \times \left(\frac{\hspace{1.5cm}}{r} \right)^2 + \pi \times \frac{\hspace{1.5cm}}{r} \times \frac{\hspace{1.5cm}}{s}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

Calculating Surface Area in Everyday Life

Example 1 - This example demonstrates one common application of surface area - how much paint is needed to paint a room or a building.

You want to paint the walls inside of your bedroom (not the ceiling or the floor). The bedroom is 4m wide and 6 m long. The walls are 2.5 m tall. The door and the window have a combined surface area of 1.8 m^2 , which you will not paint. A gallon of paint covers 37.2 m^2 . How many gallons of paint will you need to put two coats of paint on the walls? A can contains one gallon of paint. How many cans do you need to buy?
(3 cans)

1. Calculate the surface area of the wall that is 4 m wide 2.5 m high then multiply by two because there are two walls.

2. Calculate the surface area of the wall that is 6 m long and 2.5 m high long then multiply by two because there are two walls.

3. Add up those two areas then multiply by two because you're doing 2 coats of paint.

4. Subtract 1.8 because you're not painting the door and the window.

5. Divide your total by 37.2 to see how many gallons of paint you'll need.

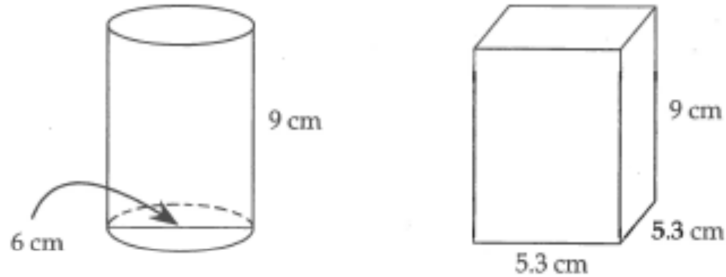
6. You can't buy partial cans of paint. How many cans do you need to buy to paint the walls (but not the door or window) with two coats of paint?

Example 2

This example compares the amount of packaging of a product by finding the surface area of the packaging.

Second Squeeze, a juice company, is trying to decide whether to sell its new juice in a can or a juice box. The company has “gone green” so it wants to use a minimal amount of packaging to reduce its waste. Which should it choose if both the can and the box hold the same amount of juice?

(can)



Try it!

Cherida has put siding on her house. Each piece of siding is 10 cm by 2 m. She used 1344 pieces of siding. Cherida did not cover the windows and doors, which have a total surface area of 7.2 m². What is the total surface area of the outer walls of Cherida's house? (276 m²)

(hint : Convert 10 cm to metres. Look at the conversion chart at the back of the booklet if you're not sure how.)

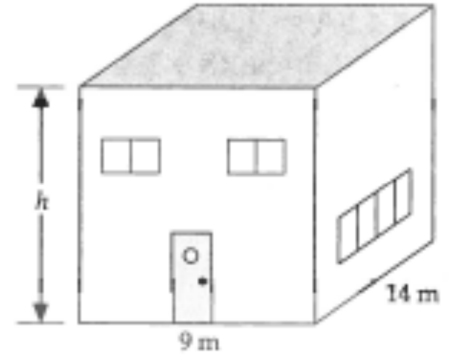


Show your work clearly. Indicate formulas, values substituted in formulas. Use titles to show what you're calculating. You'll need to find:

- area of one piece of siding
- area of 1344 pieces of siding
- total surface area which is the siding plus the windows/doors

Example 3 – Finding a missing dimension

Oliver is measuring the dimensions of the walls of Cherida's house (from p. 36). He has measured the width to be 9m and the length to be 14 m. Based on the surface area of the 4 walls, Oliver calculates the height of the house to be 12 m. Is he correct? If he is not, what is the correct answer?
(no. 6m)



(Hint: Write out the formula with the values you know, including surface area. Then use algebra to find "h".)

Use a formula for surface area that only has TWO pairs of sides (instead of three – we're only finding walls; not roof and floor.) Substitute the numbers into the formula for Length and Width and h for the Height. Substitute the surface area from the "try it" into the formula for the Surface Area.

Add like terms (add the the coefficients only of the terms with the same variable [letter])

$$276 = 46h$$

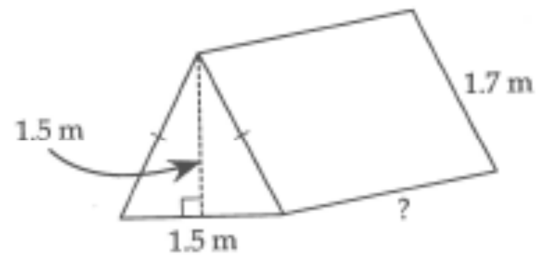
→
-46 is the coefficient – the number multiplied by h

-Divide both sides by the coefficient 46

Try it!

Rosselle knows that she has used 12.05 m^2 of fabric to make a tent, including the floor of the tent. How long is the tent from one end to the other? (1.9 m)

(Hint: Solve the question as if you didn't have the answer. Write the formula for SA of this shape. Substitute the numbers into the formula for Base and Height and Surface Area and L for the Length of the tent. Use algebra to find "L".



Add like terms – add the coefficients only of the terms with the same variable (letter)

$$12.05 = 2.25 + 5.1L$$

- 2.25 is **constant** (number) that is on the same side of the equation as the term with a variable (letter)

- subtract 2.25 from both sides

Divide both sides by the coefficient (the number multiple by L)

Composite Objects

Now that you can calculate the surface area of basic 3-D objects, you can apply these methods to composite objects. A **composite object** is an object made up of **more than one** 3-D shape.

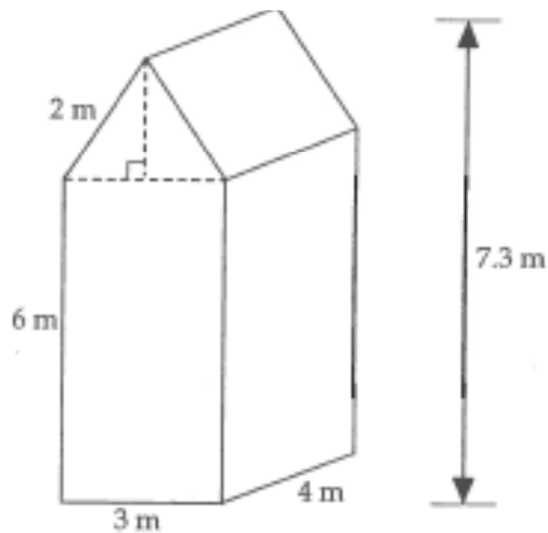
If a composite shape is made up of one 3-D shape attached to another shape, the **faces that overlap are not included** in the surface area. Either don't include them or subtract them after.

-The top face of the bottom object and the bottom face of the top object would not be included in the original calculation.

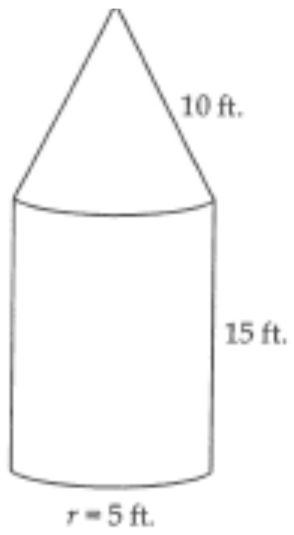
- Alternatively, you can calculate **the two shapes as per normal and then subtract the two overlapping faces that are not part of the outside surface** (the overlapping faces are 'hidden' inside the new object).

You can use the formula OR use the faces approach. Show your work clearly. Indicate formulas, values substituted in formulas. Use titles to show what you're calculating. (118 m²)

Example 1

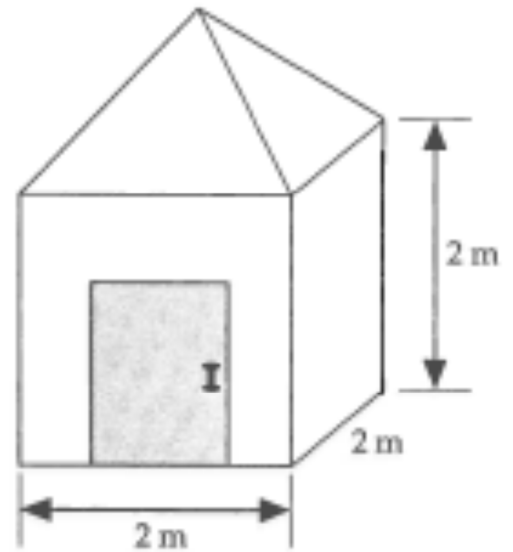


Example 2 (706.8 ft²)



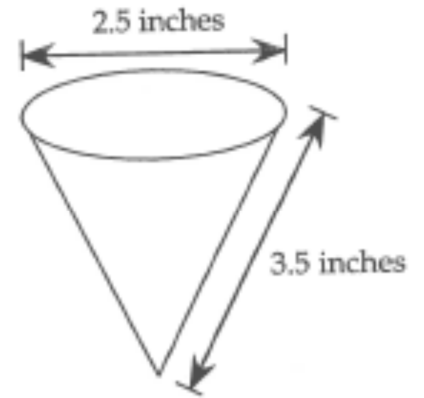
Example 3 – amount of material in a 3-D composite object

Jeremiah is building a shed. He has 20 m^2 of sheet metal that he will be using for the sides and the roof but not the door. The door is 1 m by 1.5 m . **Does he have enough sheet metal for the shed if the slant height of the roof is 1.5 m** ? (no)

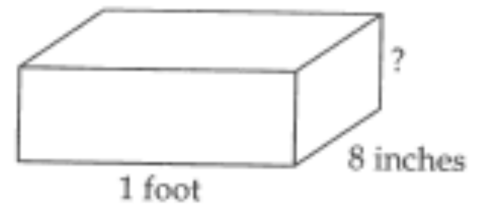


Try it!

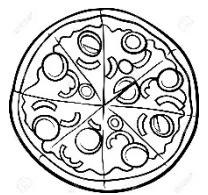
- a) How much paper do you need to make a cone paper cup? (17.5 in^2)



- b) Mac has wrapped a box using 352 in^2 , with no overlap. What is the height of the box? (hint: make all the dimensions the same unit!)
(1 foot = 12 inches) (4")



- c) Which is the larger amount of pizza: 2 small 11" pizzas, or 1 large 15" pizza?
(11" and 15" are the diameters of the pizzas). (2 small)

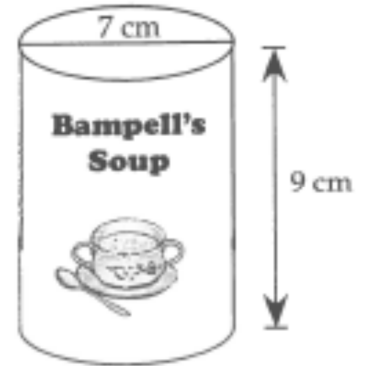


Assignment 4: Surface Area in Everyday Life

1. Bambell's Soup Company makes the labels for its cans of soup from 23 cm by 28 cm paper. If all its soup cans look like the diagram below with the same dimensions, how many labels can the company cut from one piece of paper? Is there any unused paper? Show your work. (3.yes)

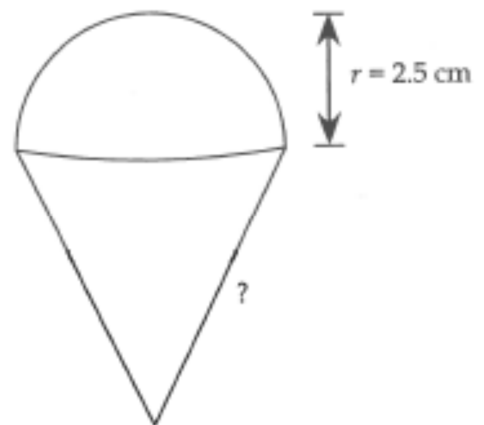
(Hint: - draw the label. Label the dimensions. How do you calculate the length of the label? *Look at p. 11 for help.*

-draw the piece of paper and sketch how many labels would fit)



2. The surface area of an ice cream cone with half a sphere (hemisphere) of ice cream is 117.8 cm^2 . The radius of the cone and ice cream is 2.5 cm. What is the slant height of the ice cream cone? Show your work. (see p. 30, 37, 38 for help) (21.6 cm)

(Hint: The circle of the half sphere and the circle of the cone are not included)



Area of 2D objects: in units²

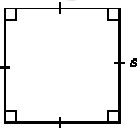

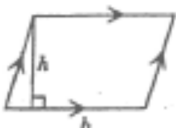
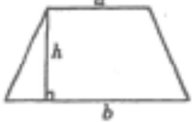
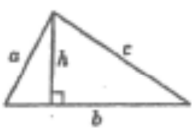

Name of Formula	Diagram	Formula
area of a square		$A = s^2$
area of a rectangle		$A = lw$
area of a parallelogram		$A = bh$
area of a trapezoid	 $A = \frac{(a+b)h}{2}$	$A = \frac{1}{2}(a+b)h$
area of a triangle	 $A = \frac{bh}{2}$	$A = \frac{1}{2}bh$
area of a circle		$A = \pi r^2$ (π button on calculator)

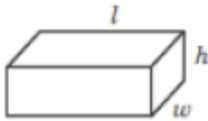

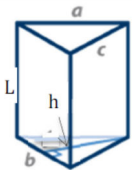
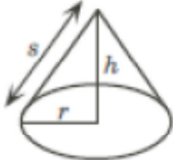
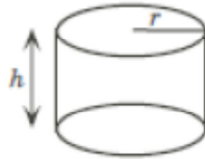
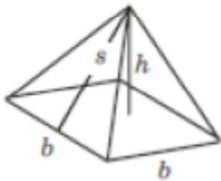
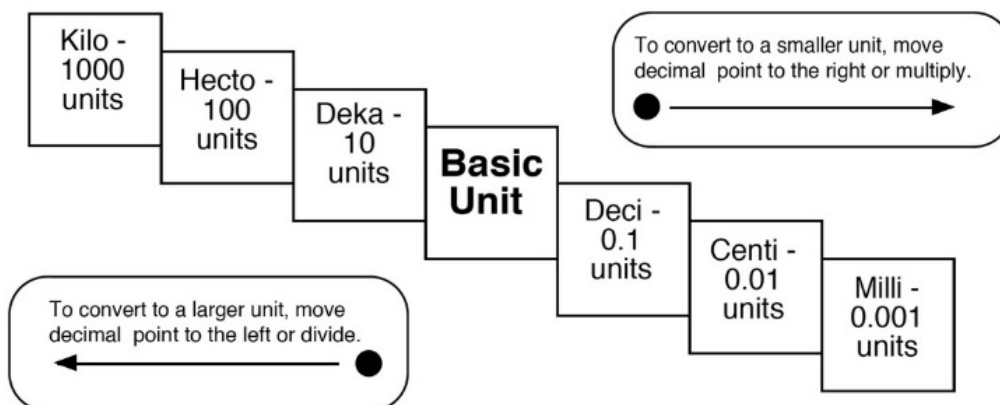
Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
rectangular prism		$SA = 2wh + 2lw + 2lh$ sides top/bottom front/back	$V = lwh$
sphere		$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
<i>Triangular Prism</i>		$SA = 2\left(\frac{bh}{2}\right) = aL + bL + cL$ bases sides (isoc. Δ - 2 sides same area)	$V = \left(\frac{bh}{2}\right)L$
cone		$SA = \pi rs + \pi r^2$ (slanted side only)	$V = \frac{1}{3}\pi r^2 h$
cylinder		$SA = 2\pi rh + 2\pi r^2$ Side bases	$V = \pi r^2 h$
square base pyramid		$SA = b^2 + 2sb$ base sides (s - slant height h - vertical height from apex to base of pyramid)	$V = \frac{1}{3}b^2 h$

1 foot = 12 inches

Metric Conversion Chart



1 km = 1000 m

1 kg = 1000 g

1 m = 100 cm

1 cm = 10 mm

1 g = 1000 mg

1 ℓ = 1000 ml

